Tunable Pendulum Energy Harvester Using Cone Continuously Variable Transmission

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***Abstract*—This paper proposes a tunable pendulum energy harvesting system incorporating a Cone Continuous Variable Transmission (Cone CVT) design. The goal is to build upon the resonance state observed in previous adaptive-tuned vibration absorber research by integrating a converter to transform mechanical energy into electrical energy. The system dynamically tunes itself in response to varying excitation frequencies. In this design, a Cone CVT is installed between the pendulum stem and the dynamo. Additionally, a conventional PID controller is employed to adjust the wheel position. A mathematical model is developed to predict system response and performance for different design parameters. MATLAB ODE45 and ODE23 solvers are utilized for simulation, generating data for dynamic system analysis. The analytical model is validated by tuning the frequency within the range of 1 to 5.24 rad/s. The outcomes demonstrate that, following the tuning algorithm, the pendulum enters a resonance state, leading to a significant increase in output voltage, approximately 10 times compared to the initial value.**

***Index Terms*—Renewable Energy, Energy Harvesting, Cone Continuous Variable Transmission, PID Controller, Auto-tuning Harvester, Tunable frequency**

I. Introduction

Energy harvesting involves the extraction of small amounts of energy from the surroundings and converting it into electrical power using various components. It is also termed as "environmental power generation." Firstly, it generates electricity from sources such as sunlight and vibrations, making it eco-friendly. Secondly, it capitalizes on the proximity to energy sources [1]. The available renewable energy sources that can be harvested are solar energy [6], thermal energy [7], and mechanical energy [8] Lately, energy harvesters utilizing pendulum systems have frequently found application in environments characterized by ultra-low frequencies, such as ocean waves, human movement, and structural vibrations. [2] In the previous study, a novel adaptive tuned vibration absorber (ATVA) design was introduced to mitigate the resonance of the primary system, potentially causing damage or sudden failure to machinery or buildings. Additionally, a cone-based continuously variable transmission (cone CVT) was proposed to incorporate a dynamic vibration absorber (DVA) capable of adjusting itself to varying excitation frequencies. [3]

The idea behind vibration energy harvesting from the pendulum using cone CVT involves adjusting the external excitation frequency to match the resonance frequency of the pendulum. The goals are to create an ideal system that can be tunable to a wide range of excitation frequencies and accomplish this tuning operation with low energy consumption and can be able to take advantage of excessive vibration which is a resonance from vibration absorber to make energy harvesting. Therefore, the automatic self-tuning technique of the energy harvester's resonance frequency is essential to sustain maximum power output.

In this paper, we propose a tunable pendulum energy harvester using cone CVT. To achieve these goals, the design described in this paper installs a cone CVT between the pendulum stem and dynamo, and another approach for tuning the position of the wheel is to use the conventional PID controller. This paper is organized as follows: In Section II problem statement. In Section 3 Methodology and PID controller design of the system. In Section 4 mathematical model. In Section 5 the results of numerical simulation are presented, while the last Section summarizes the obtained results.

II. Problem Statement

This paper explores the dynamic tuning of the natural frequency of the Adaptively Tuned Vibration Absorber (ATVA) with a Cone-based Continuously Variable Transmission (CVT) to align with the excitation frequency, resulting in excessive vibrations. This surplus energy, typically lost to the environment, presents untapped potentia1 [4]. Leveraging the resonance state from previous research on vibration absorbers [4], the study aims to harness this energy for harvesting by modifying the model and converting mechanical energy into electrical energy. However, a challenge arises from the inability to control the output voltage within physical limitations. Variations in the external excitation frequency lead to a mismatch with the resonance frequency, causing a significant drop in output power. Therefore, implementing an autonomous self-tuning technique for the resonance frequency of the energy harvester becomes essential to maintain maximum output power.

III. Methodology and PID Controller Design of the System.

In this paper's methodology, we can determine the appropriate wheel position based on the resonance point. The autonomous self-tuning technique proposed in this paper is shown in Fig. 1. This approach starts by obtaining the external excitation frequency from the sensor and computing the target position of the wheel. The resulting calculation is then relayed to the PID controller to initiate the movement of the DC motor, thereby adjusting the wheel position.

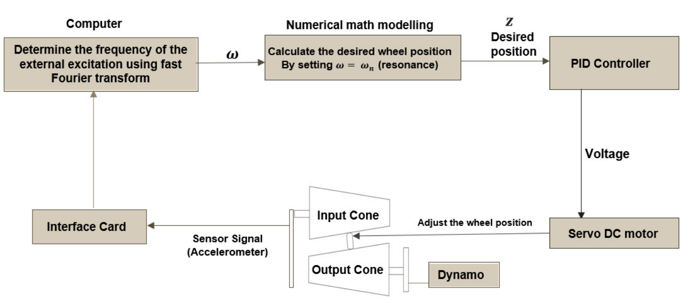

Fig. 1. The autonomous self-tuning technique diagram

The closed-loop feedback block diagram for adjusting the wheel position is illustrated in Fig. 2.

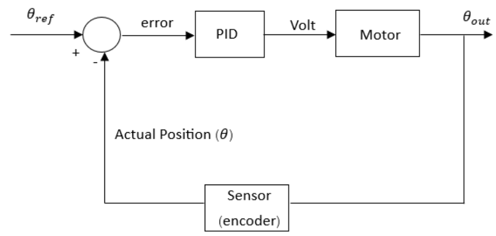

Fig. 2. the block diagram of the feedback control system for adjusting the wheel position

The real-time DC motor position control system comprises three primary components: a PID controller, responsible for regulating the position of the DC motor by adjusting the input voltage; the DC motor itself, which drives the position control mechanism; and a sensor, used to track the direction of a DC motor and the DC motor will move the ball and screw which connected to the wheel.

The Isometric view of Cone CVT is shown in Fig. 3.

IV. Mathematical Model

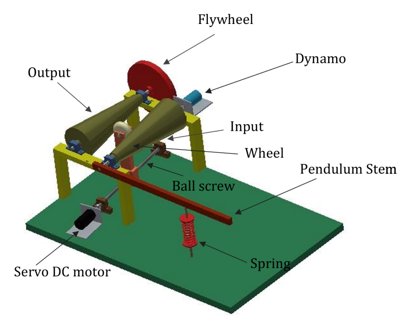

Fig. 3. Isometric view of cone CVT

In this system, the cone CVT consists of two cones and a wheel. The wheel will be positioned in the middle between the two cones. The moment will be transmitted between the input and output cone. The gear ratio is modified by varying the radius ratio between the input and output cones through adjustments in the wheel position.

The schematic of the primary structure is shown in Fig. 4. The primary structure is modeled by one spring connected vertically the pendulum stem lying horizontally, and the excitation is a direct resonance. A vertical external force stimulates the system

$$y = Ye^{i\omega t} \tag{1}$$

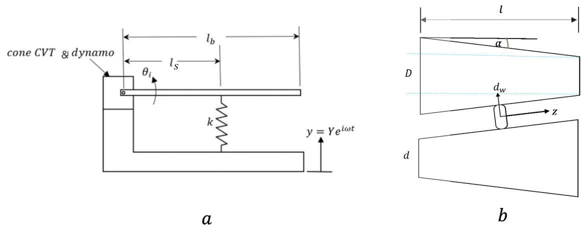

Fig. 4. The schematic of the primary structure (a) schematic of energy harvester (b) schematic of cone CVT

and the system enters the resonance state, when $\omega = \omega_n$, where ω_n is the natural frequency of pendulum- CVT system.

It is assumed that no slip occurs between cones and the wheel, therefore the gear ratio G between the input cone and the output cone can be expressed as:

$$G = \frac{\theta_o}{\theta_i} = \frac{Dl - (D-d)z\cos\alpha}{dl + (D-d)(z\cos\alpha + d_w \sin\alpha)}. \tag{2}$$

From (2) the relation between wheel position z and gear ratio of cone CVT G can be written as:

$$Z = \frac{Dl - dlG - d_w \sin\alpha G(D-d)}{(G+1)(D-d)\cos\alpha}, \tag{3}$$

where D is the bigger diameter of the cone, d is the smaller diameter of the cone, l is the length of the cone, α is the taper angle, z is the wheel position, θ_o is the angular displacement of output cone and θ_i is the angular displacement of input cone Note that the gear ratio of cone CVT G depends on the wheel position z.

The kinematic energy K of the system is

$$K \quad = \quad K_{cvt} + K_b, \tag{4}$$

where

$$K_{cvt} = \left[I_{ci}\dot{\theta}_i^2 + (I_{co} + I_{fw})\, G^2 \right] \dot{\theta}_i^2 \tag{5}$$

$$K_b = \frac{1}{2} m_b \left(\dot{y} + l_G \dot{\theta}_i \right)^2 + \frac{1}{2} I_b \dot{\theta}_i^2, \tag{6}$$

where K_{cvt} is the rotational kinematic energy, K_b is the kinematic energy of the pendulum stem, I_{ci} is the moment of inertia of input cone, I_{co} is the moment of inertia of output cone, I_{fw} is the moment of inertia of the flywheel, K_b is the kinematic energy of pendulum stem, I_b is the moment of inertia of pendulum stem, l_G is the length between rotational point and center of mass and $\dot{\theta}_i$ is the angular velocity of input cone. The potential energy of the system will be:

$$V = \frac{1}{2} k \left(l_s \theta_i \right)^2, \tag{4}$$

when k is spring constant.

From Kirchoff's circuit laws, we have the equation of an equivalent circuit as

$$k_e \dot{\theta}_i - \mathrm{L}\frac{di}{dt} + (R_L + R_g)\, i = 0. \tag{7}$$

The Euler-Lagrange equation of this system is given by

$$L = K - V. \tag{8}$$

From this equation, we obtain

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} = \tau_i, \tag{9}$$

where the torque τ_i created by the current flowing through the coil can be expressed as:

$$\tau_i = -c\dot{\theta}_i - k_\tau i \tag{10}$$

and k_τ is the torque constant.

Then the motion for the energy harvester (harvesting system) is

$$\begin{aligned} &[I_b + J_g + 2I_{ci} + 2\,(I_{co} + I_{fw})]\, G^2 + m_b l_G^2]\ddot{\theta}_i \\ &\quad + c\dot{\theta}_i + k_\tau i + k l_s \theta_i = -m_b l_G \ddot{y}. \end{aligned} \tag{11}$$

The natural frequency ω_n of the system is

$$\omega_n = \sqrt{\frac{k l_s}{I_b + J_g + 2I_{ci} + 2\,(I_{co} + I_{fw})\, G^2 + m_b l_G^2}}. \tag{12}$$

Thus, the natural frequency of the energy harvester can be adjusted in real-time by adjusting its wheel position.

Electromagnetic harvesters operate at low frequencies, rendering the inductance of the generator negligible in comparison to its impact on the electric circuit. Therefore, the equation for the equivalent circuit can be written as:

$$P_L = i^2 R_L = R_L \left(\frac{k_e \dot{\theta}_i}{R_L + R_g} \right)^2. \tag{13}$$

We get the equation of this system

$$\begin{aligned} &\left[[I_b + J_g + 2I_{ci} + 2\,(I_{co} + I_{fw})]\, G^2 + m_b l_G^2 \right] \ddot{\theta}_i + c\dot{\theta}_i \\ &\quad + \frac{k_\tau k_e}{R_L + R_g}\dot{\theta}_i + k l_s \theta_i = -m_b l_G \ddot{y} \end{aligned} \tag{14}$$

The average electricity power at the resonance state can be written as

$$P_{\omega=\omega_n}^{avg} = \frac{1}{2}\left(\frac{m_b l_G \omega_n^2 Y}{\frac{C\sqrt{R_L}}{k_t} + \frac{k_t}{\sqrt{R_L}}} \right)^2. \tag{15}$$

TABLE I
PARAMETERS USED FOR SIMULATION

Symbol	Value	Units
$I_{ci} = I_{co}\ [kg \cdot m^2]$	0.00475	Moment inertia of input and output cone
$I_{fw}\ [kg \cdot m^2]$	0.005	Moment inertia of flywheel
D [mm]	120	The bigger diameter of cone CVT
d [mm]	40	The smaller diameter of cone CVT
l_c [mm]	300	Length of cone CVT
d_w [mm]	74	The diameter of the wheel
α	7.594	Taper angle
k [N/m]	100	Spring constant
m_b [kg]	2	Pendulum stem mass
l_b [m]	0.4	Length of pendulum stem
$I_b\ [kg \cdot m^2]$	0.0266	Moment inertia of pendulum stem
l_s[m]	0.2	Length from rotational point and spring
l_G [m]	0.2	Length from rotational point and center of mass
c	0.028	Mechanical damping
k_τ [N/A]	0.5	Torque constant
$J_G\ [kg \cdot m^2]$	$83\ x\ 10^{-7}$	Shaft inertia
R_m [Ω]	4	Armature resistance
$k_e\ [V \cdot rad^{-1} \cdot s^{-1}]$	0.5	Electromotive force constant
L [H]	$2.75x\ 10^{-6}$	Armature inductance
$b_m\ [Nm^{-1} \cdot rad \cdot s^{-1}]$	$3.508\ x\ 10^{-6}$	Motor damping

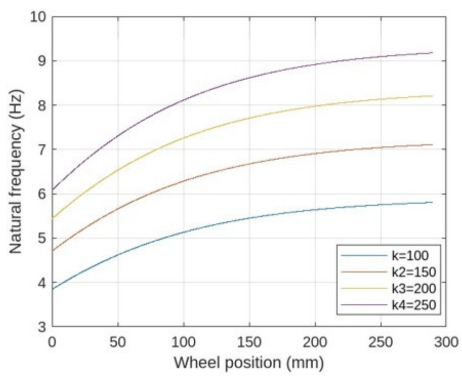

Fig. 5. The natural frequencies of the system versus wheel position under different spring constant

V. NUMERICAL SIMULATION

The simulation parameters are shown in Table I. Numerical simulation is used to model and analyze complex systems.

The natural frequencies ω_n of the system versus wheel position z under different spring constant k is shown in Fig. 5. From Fig. 5, during the experiments, the spring constant can be adjusted by changing the characteristic of spring with different spring constant. The obtained results show the natural frequency of the system increases with the increase of the wheel, moreover, increasing the spring constant increases the tunable frequency range. In addition, this parameter is more effective at the low-frequency range.

The average power versus frequency under different wheel positions is shown in Fig. 6. From Fig. 6, after tuning the frequency of external excitation to match the natural frequency of the system into the resonance state the results show the highest average power could be generated at the resonance state. The increase of wheel position at the resonance state, the highest of the average power could be made.

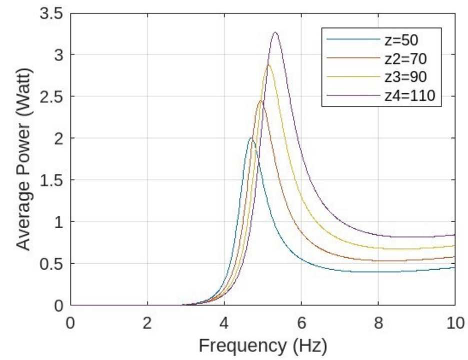

Fig. 6. The average power versus frequency under different wheel positions

The output voltage and wheel position responses of the energy harvesting system to base excitation using an autonomous self-tuning technique and a PID controller that repositions the wheel in real time are depicted in Fig. 7 and Fig. 8, respectively. As depicted in Fig. 7, at the initial stage, the energy harvester is excited with a frequency of 1 rad/s. In Fig. 8, the initial position of the pendulum is 50 mm. This position is chosen such that the external excitation frequency does not match the natural frequency of the energy harvester. As the system evolves over 20 seconds, the frequency of the external excitation is changed to 5.34 rad/s. Simultaneously, the PID control system begins adjusting the position of the wheel automatically through a DC servo motor. After 22 seconds, the pendulum enters a resonance state, causing the output voltage to increase significantly, approximately 10 times higher than the initial value.

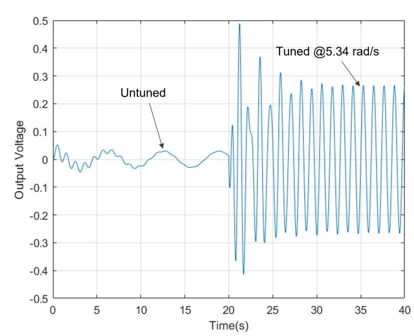

Fig. 7. Time response of the closed-loop system output voltage

VI. CONCLUSION

In this paper, the pendulum was directly coupled to the cone CVT for automatic natural frequency adjustment—the numerical approach used to test the proposed energy harvester. The effect of different spring constant parameters (k) on the energy

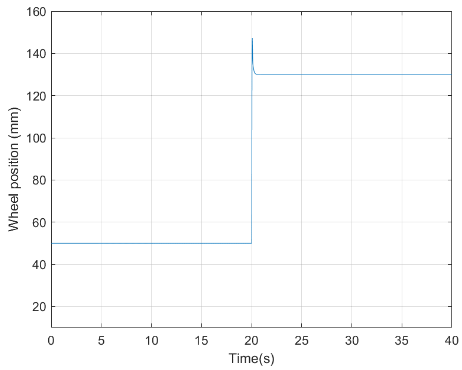

Fig. 8. Time response of the closed-loop system wheel position

harvesting by adjusting spring characteristic, and another was changing the external excitation frequency to match the natural frequency, the result shows that after changing the wheel position the increasing of the wheel position at the resonance state, the highest of the average power could be made. From the autonomous self-tuning technique at t = 20 s, the external excitation frequency changes suddenly. The harvester can automatically adjust its natural frequency to match the change in external excitation frequency.

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